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Thermodynamics of Squeezed State for Mesoscopic RLC Circuits

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Abstract We present a density matrix of a mesoscopic RLC circuits to make it possible to analyze the connection between the initial condition and the certain temperature. Our results show that the quantum state evolution will be closely related to the initial condition; the system evolves to generalized coherent state if it is in ground state initially, and evolves to squeezed state if it is in excited state initially. In addition, we also obtain squeezed minimum uncertainty state with satisfying certain condition in mesoscopic RLC circuit.

Keywords Mesoscopic RLC circuit · The maximum entropy principle; Evolution of quantum state

1 Introduction

It is well known that many physics problems in various fields such as quantum measurement, quantum computations, and quantum optic theory are related to investigating the property of mesoscopic circuits. And the research on quantum effects of electric circuits will be helpful to the miniaturization of integrate circuits and electric components. In recent years mesoscopic system have been widely studied. Many quantum effects have been revealed [1–3], of which the most significant one is the phase coherent tunneling of single electron. Some non-classical states, such as coherent states, squeezed states [4], etc., have been widely studied. Since the squeezed states can achieve lower quantum noise than the zero point fluctuations

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associated with vacuum or coherent states, they provide a way to manipulate quantum fluctuations and offer a promising future in different applications such as optical communication and gravitational wave detection. In the context of condensed matter physics, the idea of squeezed effects has also been used squeezed phonon states have been observed [5]. In fact, squeezed states also exist in mesoscopic circuits. The aspects interests people: nanometer electronic elements acting as quantum dot or quantum line, nonclassical quantum effect in mesoscopic quantum circuits, and electronic approaches to quantum state. Compared to optic approaches, electronic ones do not restraint to too rigorous macroscopic condition. The existing research results indicate that, under some certain initial condition and operation, the coherence state, squeezed coherence state, squeezed coherence state, and the squeezed minimum uncertainty state etc. can be obtained in mesoscopic circuits.

Practically, a circuit system always works under some certain temperature. Thus, the application if thermodynamics to electric circuits has a long and fruitful history. In the late 1920's, the second law-Nyquist the spectrum of the fluctuation force acting in an equilibrium electric circuit was deduced by applying general principles of thermodynamic, in particular. The result known as the Nyquist spectrum, was later confirmed by microscopic approaches. Nearly 20 years later, the second law was applied to analyze a circuit containing rectifying elements by Brillouin. In the context of electric circuits, Landauer considered another formulation of the second law, i.e., the Clausius inequality. Then, Stratonovich analyzed the equilibrium thermodynamics of linear an nonlinear circuits thoroughly. In fact, quantum statistic mechanics, which is based in quantum mechanics, is applied to study the physical qualities of mesoscopic circuits. Therefore, how to construct a density matrix becomes a key point since the density matrix of a system is difficult to calculate. In thermo-field-dynamics [6], it can be said that the squeezed states, the different "knowledge" introduced in a general density matrix and the Hamiltonian dynamics.

In this paper, we discuss dynamical behavior of evolution quantum state of mesoscopic circuit with dissipation. The paper is organized as follows. A brief resume of the maximum entropy principle concepts is given in Sect. 2. Our approach to coherent and squeezed states is of a mesoscopic RLC circuit using a statistical operator is developed in Sect. 3. Section 4 is devoted to study of squeezing in the mesoscopic circuit with constant frequency. Finally, some conclusions are drawn in Sect. 5.

2 The Maximum Entropy Principle

The maximum entropy principle approach provides us with a definite prescription to construct the density matrix $\hat{\rho}$, starting from the knowledge of the expectation values of, say M, and operators $\hat{O}_j$ ($\hat{O}_0$ is identity operator) are defined as usual,

$$\langle \hat{o}_j \rangle = \mathrm{Tr}(\hat{\rho}\hat{O}_j), \tag{1}$$

$j = 0, 1, \dots, M$. The identity operator is included in the original set to fulfill the normalization condition $\mathrm{Tr}\,\hat{\rho} = 1$. The MEP version of $\hat{\rho}$ reads

$$\hat{\rho} = \exp\left[-\lambda_0 \hat{I} - \sum_{j=1}^{M} \lambda_j \hat{O}_j\right], \tag{2}$$

where λ_j are $M+1$ Lagrange multipliers, which are determined to satisfy (1) and the maximization of the entropy $S[\hat{\rho}]$ (taking the Boltzmann constant equal to unity),

$$S[\hat{\rho}] = -\operatorname{Tr}(\hat{\rho}\hat{O}_j) = \lambda_0 + \sum_{j=1}^{M} \lambda_j \langle \hat{O}_j/\hat{\rho} \rangle. \tag{3}$$

It can be proven that, since $\hat{\rho}(t)$ obeys the Liouville equation and (3), the set if operators $\hat{Q}_j$ originally considered must be eventually extended to a set of q relevant operators selected in order to close a partial Lie algebra under commutation with the Hamiltonian $\hat{H}$

$$[\hat{H}, \hat{O}_k] = i\hbar \sum_{j=0}^{q} \hat{O}_j g_{jk}, \tag{4}$$

where the g_{jk} are the element (c numbers) of a $q \times q$ matrix G (which may depend upon the time if $\hat{H}$ is time dependent). The closure condition given by (4) allows us to solve the dynamical problem using a set of coupled equations for the λ_js (which is formally equivalent to the time-dependent Schrodinger equation)

$$\frac{d\lambda_j}{dt} = \sum_{k=0}^{q} g_{jk}\lambda_k. \tag{5}$$

Analogously, Ehrenfest's theorem also makes it possible to obtain the temporal evolution of the expectation values of the relevant operators. On the condition that $\hat{Q}_j$ does not depend on explicitly upon the time, the result is

$$\frac{d\langle \hat{O}_j \rangle}{dt} = -\sum_{k=0}^{q} \langle \hat{O}_k \rangle g_{kj}, \tag{6}$$

$j = 0, 1, \ldots, q$.

3 Density Matrix for a Mesoscopic RLC Circuit

The classical kinetic equation of a RLC circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = \varepsilon(t),$$

where L, C, and R stand for the inductance, capacitance, and resistance, respectively. $\varepsilon(t)$ is the external voltage source. $q(t)$ is charge being regarded as a coordinate of the system. The generalized current is defined as $p(t) = L\dot{q}$. It is well known that the resistance of a circuit produced by the collisions between crystal lattice and electrons. So, in fact, the mesoscopic RLC circuit can be equal to an interactive system of an electromagnetic harmonic oscillator and the environment bath of the lattice oscillators [7]. It can be easy to proven that the coordinate $\hat{q}$ and momentum $\hat{p}$ of a mesoscopic dissipative circuit are a pair of linear Hermitian operators, satisfying the commutation relation $[\hat{q}, \hat{p}] = i\hbar$ [8]. Therefore, the Hamiltonian of the circuit without source is given by [9]

$$\hat{H} = \frac{1}{2L}\hat{p}^2 + \frac{1}{2C}\hat{q}^2 + \frac{\alpha}{2}(\hat{q}\hat{p} + \hat{p}\hat{q}), \tag{7}$$

$\alpha = 2R/L$. The Hamiltonian of the system is equivalent to a harmonic oscillator, which plus a term proportional to $2\hat{N} = (\hat{q}\hat{p} + \hat{p}\hat{q})$, as it is constructed by inducing the three operators into SU(1, 1) (one of the groups associated with: $(\hat{q}^2, \hat{p}^2, \hat{N})$). Defining the annihilation and creation operators $\hat{a}$ and $\hat{a}^+$ by the relations

$$\hat{q} = \sqrt{\frac{\hbar}{2L\omega_0}}(\hat{a}^+ + \hat{a}),$$

$$\hat{p} = \sqrt{\frac{\hbar L\omega_0}{2}}(\hat{a}^+ - \hat{a}).$$

Then, the Hamiltonian of the circuit reads

$$\hat{H} = \hbar\omega_0\left(\hat{a}^+\hat{a} + \frac{1}{2}\right) + i\frac{\alpha\hbar}{2}(\hat{a}^{+2} - \hat{a}^2)$$

with $[\hat{a}, \hat{a}^+] = 1$, and $\omega_0 = \frac{1}{\sqrt{LC}}$.

Using the following transformation, we redefine the creation and annihilation operators:

$$\hat{A} = \sqrt{\frac{1}{2L\hbar\Omega}}[L(\Omega + i\alpha)\hat{q} + i\hat{p}], \tag{8}$$

$$\hat{A}^+ = \sqrt{\frac{1}{2L\hbar\Omega}}[L(\Omega - i\alpha)\hat{q} - i\hat{p}], \tag{9}$$

and get the Hamiltonian

$$\hat{H}_1 = \hbar\Omega\left(\hat{A}^+A + \frac{1}{2}\right), \tag{10}$$

with

$$\Omega^2 = \omega_0^2 - \alpha^2.$$

In the following sections, we will study dynamic behavior of charge or current and the presence of squeezed states in the mesoscopic RLC circuit. Equation (7) shows that the movement of the system is equivalent to that of a quantum damp harmonic oscillator. The operators of the system closes a semialgebra, i.e., $(\hat{q}, \hat{p})$, or with SU(1, 1), i.e., $(\hat{q}^2, \hat{p}^2, \hat{N})$. Thus the operators included in these algebras can be naturally selected as relevant operators. However, in order to obtain states with a very special fluctuations relation between $\hat{q}$ and $\hat{p}$, we should consider information regarding not only these operators, but also their squares.

The relations among the initial conditions of the relevant operators and the initial values of the Lagrange multipliers cannot be set arbitrarily. The MEP prescribes that this can be made using (8) and (9). The density matrix for this problem is given by

$$\hat{\rho} = \exp(-\lambda_0\hat{I} - \lambda_1\hat{q} - \lambda_2\hat{p} - \lambda_3\hat{q}^2 - \lambda_4\hat{p}^2 - \lambda_5\hat{N}). \tag{11}$$

For the sake of convenience, we define a new creation operator

$$\hat{b}^+ = |\cosh r|e^{-i\varphi}\hat{A}^+ + |\sinh r|e^{-i\theta}\hat{A} + |\gamma|, \tag{12}$$

with $[\hat{b}, \hat{b}^+] = 1$. In (12), r represents the squeezed parameter. The inverse is (12) is

$$\hat{A}^+ = |\cosh r|e^{-i\varphi}\hat{b}^+ - |\sinh r|e^{-i\theta}\hat{b} + |\gamma|[|\sinh r|e^{-i\theta} - |\cosh r|e^{-i\varphi}].$$

In order to get the diagonalization, one should include the Hamiltonian as a relevant operator. Therefore, the density matrix reads

$$\hat{\rho} = \exp(-\lambda_0\hat{I} - \lambda_1\hat{q} - \lambda_2\hat{p} - \lambda_3\hat{q}^2 - \lambda_4\hat{p}^2 - \lambda_5\hat{N} - \beta\hat{H}_1). \tag{13}$$

Using the normalization condition, it is easy to prove that the density matrix can be written in terms of the new operators $\hat{b}^+$ and $\hat{b}$ as

$$\hat{\rho} = 2\sinh\left(\frac{\beta\Omega}{2}\right)\exp\left(-\frac{\beta\Omega}{2}\right)\exp(-\beta\Omega\hat{b}^+\hat{b}), \tag{14}$$

if the Lagrange multipliers verify

$$\frac{1}{\sqrt{2\Omega}}\lambda_1 + \lambda_{2g}\sqrt{\frac{\Omega}{2}} = \beta\Omega|\gamma|[|\sinh r|e^{-i\theta} + |\cosh r|e^{-i\varphi}], \tag{15}$$

$$\frac{1}{\sqrt{2\Omega}}\lambda_1 + \lambda_{2g^*}\sqrt{\frac{\Omega}{2}} = \beta\Omega|\gamma|[|\sinh r|e^{i\theta} + |\cosh r|e^{i\varphi}], \tag{16}$$

$$\left(\frac{1}{\Omega}\lambda_3 + |g|^2\Omega\lambda_4 - \mathrm{Re}(g)\lambda_5\right) = 2\beta\Omega(\sinh r)^2, \tag{17}$$

$$\mathrm{Re}\left(\frac{1}{\Omega}\lambda_3 + g^2\Omega\lambda_4 - g\lambda_5\right) = 2\beta\Omega|\sinh r||\cosh r|\cos(\theta+\varphi), \tag{18}$$

$$\mathrm{Im}\left(\frac{1}{\Omega}\lambda_3 + g^2\Omega\lambda_4 - g\lambda_5\right) = -2\beta\Omega|\sinh r||\cosh r|\sin(\theta+\varphi), \tag{19}$$

with $g = \alpha\Omega^{-1} + i$ and $\hbar$ was taken equal to 1 in followings. So, from (1) and (15), (19), we obtain their expectation values

$$\langle\hat{q}\rangle = \frac{2|\gamma|}{\sqrt{2\Omega}}(|\sinh r|\cos\theta - |\cosh r|\cos\varphi),$$

$$\langle\hat{p}\rangle = |\gamma|\sqrt{2\Omega}\left[-|\sinh r|\left(\frac{\alpha}{\Omega}\cos\theta - \sin\theta\right) + |\cosh r|\left(\frac{\alpha}{\Omega}\cos\varphi - \sin\varphi\right)\right],$$

$$\langle\Delta\hat{q}\rangle^2 = \frac{1}{2\Omega}\left(1 + \frac{2\lambda_4}{\beta}\right)\coth\left(\frac{\beta\Omega}{2}\right), \tag{20}$$

$$\langle\Delta\hat{p}\rangle^2 = \frac{\Omega}{2}\left(1 + \frac{\alpha^2}{\Omega^2} + \frac{2\lambda_3}{\beta\Omega^2}\right)\coth\left(\frac{\beta\Omega}{2}\right), \tag{21}$$

$$\langle\Delta\hat{q}\rangle\langle\Delta\hat{p}\rangle = \frac{1}{2}\left[1 + \frac{(\alpha+\lambda_5)^2}{\Omega^2}\right]\coth\left(\frac{\beta\Omega}{2}\right). \tag{22}$$

So the mean values have an explicit temperature dependence given by the factor $\coth(\beta\Omega/2)$. With all these elements at hand we can analyze the quantum state evolution and the possibility of obtaining squeezing in charge or current of the mesoscopic RLC circuit.

4 Evolution of Quantum State for Mesoscopic RLC Circuit

In this section, we will study the mesoscopic RLC circuit. Using the Heisenberg equation of a quantum mechanical operator, we know that the set of relevant operators of the circuit is the operators also close semialgebra under commutation with H_1 yielding

$$G_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & \omega_0^2 \\ 0 & -1 & \alpha \end{pmatrix}.$$

Similarly, $(\hat{Q}^2, \hat{P}^2, \hat{N})$ closes the SU(1, 1)

$$G_2 = \begin{pmatrix} -2\alpha & 0 & \omega_0^2 \\ 0 & 2\alpha & -1 \\ -2 & 2\omega_0^2 & 0 \end{pmatrix}.$$

From (5), we can easily obtain

$$\frac{d\lambda_1}{dt} = -\alpha\lambda_1 + \omega_0^2\lambda_2, \tag{23}$$

$$\frac{d\lambda_2}{dt} = -\lambda_1 + \alpha\lambda_2, \tag{24}$$

$$\frac{d\lambda_3}{dt} = -2\alpha\lambda_3 + \omega_0^2\lambda_5, \tag{25}$$

$$\frac{d\lambda_4}{dt} = 2\alpha\lambda_4 - \lambda_5, \tag{26}$$

$$\frac{d\lambda_5}{dt} = -2\lambda_3 + 2\omega_0^2\lambda_4. \tag{27}$$

Solving the above formula, we obtain

$$\lambda_1(t) = \lambda_1(0)\cos(\Omega t) + \Omega^{-1}[-\alpha\lambda_1(0) + \omega_0^2\lambda_2(0)\sin(\Omega t)], \tag{28}$$

$$\lambda_2(t) = \lambda_2(0)\cos(\Omega t) + \Omega^{-1}[-\alpha\lambda_2(0) + \omega_0^2\lambda_1(0)\sin(\Omega t)], \tag{29}$$

$$\lambda_3(t) = \lambda_3(0) + [\lambda_3(0) - L^2\omega_0^2\lambda_4(0)]\sin^2(\omega_0 t) + \frac{L\omega_0}{2}\lambda_5(0)\sin(2\omega_0 t), \tag{30}$$

$$\lambda_4(t) = \lambda_4(0) - \frac{1}{L\omega_0}\left[\frac{\lambda_3(0)}{L\omega_0} - L\omega_0\lambda_4(0)\right]\sin^2(\omega_0 t) - \frac{\lambda_5(0)}{2L\omega_0}\sin(2\omega_0 t), \tag{31}$$

$$\lambda_5(t) = \lambda_5(0)\cos(2\omega_0 t) - \left[\frac{\lambda_3(0)}{L\omega_0} - L\omega_0\lambda_4(0)\right]\sin(2\omega_0 t). \tag{32}$$

It can be proved, form (29–32), that

$$M = [\lambda_3(t) + \omega_0^2\lambda_4(t) - \alpha\lambda_5(t)] = [\lambda_3(0) + \omega_0^2\lambda_4(0) - \alpha\lambda_5(0)]$$

is a time-dependent invariant of motion. Further, combining (17), we see that r is also an invariant related to circuit parameters and initial conditions. From (29), (31), we easily get

$$\theta + \varphi = \theta_0 + \varphi_0 - 2\Omega t. \tag{33}$$

In the following discussion, the above results and density matrix of the system are applied to study the quantum state evolution of the mesoscopic RLC circuit.

Firstly, let us consider the case that the knowledge about the system at $t = 0$ is restraint to $\langle\hat{q}(0)\rangle$, $\langle\hat{p}(0)\rangle$, β. Then, the expectation values $\langle\hat{q}^2(0)\rangle$, $\langle\hat{p}^2(0)\rangle$ and $\langle\hat{N}(0)\rangle$ should be obtained using (4), which means that we can get the same values of $\langle\hat{q}^2(0)\rangle$, $\langle\hat{p}^2(0)\rangle$ and $\langle\hat{N}(0)\rangle$ through the method of quantum mechanics or that of quantum statistic mechanics. So, the information theory formalism determines that the density matrix is

$$\hat{\rho} = \exp(-\lambda_0\hat{I} - \lambda_1\hat{q} - \lambda_2\hat{p} - \beta\hat{H})$$

i.e., $\lambda_3(0) = 0$, $\lambda_4(0) = 0$, and $\lambda_5(0) = 0$. From (25) and (26), we obtain

$$\langle\Delta\hat{q}\rangle^2 = \frac{1}{2\Omega}\coth\left(\frac{\beta\Omega}{2}\right), \tag{34}$$

$$\langle\Delta\hat{p}\rangle^2 = \frac{\Omega}{2}\coth\left(\frac{\beta\Omega}{2}\right), \tag{35}$$

and

$$S = -\ln\left[2\sinh\left(\frac{\beta\Omega}{2}\right)\right] + \frac{\beta\Omega}{2}\coth\left(\frac{\beta\Omega}{2}\right).$$

Because $\lambda_3(0) = 0$, $\lambda_4(0) = 0$, and $\lambda_5(0) = 0$, from (17), (19) we conclude that this is equivalent to taking $r = 0$ (and this condition will remain for all as is an invariant). The transformation given in (12) becomes

$$\hat{b}^+ = e^{i\varphi}\hat{A}^+ + |\gamma|.$$

We define a generalized coherent state as follows: we consider a mixed set of initial conditions, i.e., some information related to the mean values $\langle\hat{q}(0)\rangle$, $\langle\hat{p}(0)\rangle$ (extensive variables), and some with the intensive variables ($T = 0$ K). As the density matrix, in terms of the new operators $\hat{b}^+$ and $\hat{b}$, corresponding to the canonical one, therefore, at $T = 0$ K the new Hamiltonian, i.e., $\hbar\Omega(\hat{b}^+\hat{b} + \frac{1}{2})$, should condense to its ground state $|0\rangle_b$ for which

$$\hat{b}|0\rangle_b = (e^{-i\varphi}\hat{a} + |\gamma|)|0\rangle_b = 0. \tag{36}$$

So this state is an eigenstate of $\hat{A}$ (with eigenvalue $-|\gamma|$ and corresponds to $r = 0$) and this is one of the definitions of the coherence states. It is important to note that this definition of coherent state is the same as the one given by Dodonov and Man'ko. As expected, when

$\alpha = 0$ at zero temperature, the generalized coherent state is classical coherent state. And it is easily seen that the entropy is zero and we get a pure state with minimum uncertainty. The above results show that mesoscopic RLC circuits, which initially is in a ground state, will evolve to coherent state under the effect of Hamiltonian, i.e., $\hbar\Omega(\hat{b}^+\hat{b} + \frac{1}{2})$, with neither charge nor current squeezed [10]. Although it is not a minimum uncertainty state, since now

$$\langle \Delta \hat{q}\rangle\langle \Delta \hat{p}\rangle = \frac{1}{2}\left(1 + \frac{\alpha^2}{\Omega^2}\right)\coth\left(\frac{\beta\Omega}{2}\right).$$

In conclusion, the quantum state of the mesoscopic RLC circuit evolves to a generalized coherent state from the initial ground state. And the quantum fluctuation of charge and current will increase with the rising of temperature.

Secondly, if we know, at $t = 0$, the mean values $\langle \hat{q}^2(0)\rangle$, $\langle \hat{p}^2(0)\rangle$, and $\langle \hat{N}(0)\rangle$, and when these mean values are different form the ones that would be obtained by applying (4), with $\hat{\rho} = \exp(-\lambda_0\hat{I} - \lambda_1\hat{q} - \lambda_2\hat{p} - \beta\hat{H})$, this implies, in the information theory formalism, that new information is to be included in $\hat{\rho}$ by letting $\lambda_3 \neq 0$ or $\lambda_4 \neq 0$ or $\lambda_5 \neq 0$. In fact, it means the system lies in exciting state. We shall obtain a completely different dynamical behavior. From (17), (19), we obtain

$$\lambda_3(t) = \beta\omega_0^2[(\sinh r)^2 + |\sinh r||\cosh r|\cos(2\Omega t - \theta_0 - \varphi_0 + 2\xi)], \tag{37}$$

$$\lambda_4(t) = \beta[(\sinh r)^2 - |\sinh r||\cosh r|\cos(2\Omega t - \theta_0 - \varphi_0)], \tag{38}$$

$$\lambda_5(t) = 2\beta[\alpha(\sinh r)^2 + \omega_0|\sinh r||\cosh r|\cos(2\Omega t - \theta_0 - \varphi_0 + \xi)], \tag{39}$$

with

$$\sin\xi = \frac{\alpha}{\omega_0}, \qquad \cos\xi = \frac{\Omega}{\omega_0}.$$

It is important to emphasize that, in this case, the squeezing parameter $r \neq 0$ and is constant during the whole dynamical evolution. The quantum fluctuations of charge and current of the mesoscopic RLC circuit are

$$\langle \Delta \hat{q}\rangle^2 = \frac{1}{2\Omega}[\cosh 2r - |\sinh 2r|\cos(2\Omega t - \theta_0 - \varphi_0)]\coth\left(\frac{\beta\Omega}{2}\right),$$

$$\langle \Delta \hat{p}\rangle^2 = \frac{\Omega}{2}[\cosh 2r + |\sinh 2r|\cos(2\Omega t - \theta_0 - \varphi_0 + 2\xi)]\coth\left(\frac{\beta\Omega}{2}\right).$$

At $T = 0$ K, we discuss mainly the dynamical behavior of the mesoscopic RLC circuit. We can always find that when $t_1 = (2\Omega)^{-1}(\theta_0 + \varphi_0)$ satisfying

$$(\sinh r)^2 \leq |\sinh r||\cosh r|.$$

For any initial condition, charge is squeezed. And $t_2 = (2\Omega)^{-1}(\pi + \theta_0 + \varphi_0 - 2\xi)$ satisfying

$$\omega_0^2(\cosh 2r - |\sinh 2r|) \leq \Omega^2,$$

only if the initial condition satisfy

$$\frac{\lambda_3(0) + \omega_0^2\lambda_4(0) - \alpha\lambda_5(0)}{\beta} > \frac{\alpha^4}{2\omega_0^2}, \tag{40}$$

current is squeezed. From the above analysis, we can indicate that the system will be in a squeezed state of charge for any initial condition, while it will be in a squeezed state of current only when (39) was satisfied.

Ji et al. discussed squeezed and antisqueezing effects of the charge and current of non-dissipative mesoscopic LC circuit [11]. The results show that, by keeping the frequency of the mesoscopic circuit constant and its parameter varying with the step function, the quantum state of the circuit state of the circuit evolve to the squeezed coherent state from the initial vacuum state through coherent state, then the squeezed minimum uncertainty state of charge or current is generated. Now we analyze the possibility of obtaining states characterized by a minimum uncertainty product, i.e.,

$$\langle \Delta \hat{q} \rangle \langle \Delta \hat{p} \rangle = \frac{1}{2}.$$

So, from (37), when $\cos(2\Omega t - \theta_0 - \varphi_0 + \xi) = -1$ and

$$|\tanh 2r| = \frac{\alpha}{\omega_0}.$$

The quantum state of the mesoscopic RLC circuit evolves to the squeezed minimum uncertainty state.

Under a definite temperature, the quantum fluctuation of charge and current will increase when temperature rise. Undoubtedly, from (20) and (21), we know that the minimum quantum fluctuation of charge and current are respectively

$$\langle \Delta \hat{q} \rangle^2 = \frac{1}{2\Omega} e^{-2r} \coth\left(\frac{\beta\Omega}{2}\right),$$

$$\langle \Delta \hat{p} \rangle^2 = \frac{\omega_0^2}{2\Omega} e^{-2r} \coth\left(\frac{\beta\Omega}{2}\right).$$

When

$$e^{2r} > \coth\left(\frac{\beta\Omega}{2}\right),$$

i.e., the initial condition satisfies

$$\frac{\lambda_3(0) + \omega_0^2 \lambda_4(0) - \alpha\lambda_5(0)}{\beta} > 2\Omega^2 \left(\sinh\left\{-\frac{1}{2}\ln\left[\tanh\left(\frac{\beta\Omega}{2}\right)\right]\right\}\right)^2,$$

the classical quantum squeezing effect of current or charge appears. In the same way as for $\hat{q}$, we obtain that there will be squeezing in $\hat{p}$ for any initial condition satisfying

$$\frac{\lambda_3(0) + \omega_0^2 \lambda_4(0) - \alpha\lambda_5(0)}{\beta} > 2\Omega^2 \left(\sinh\left\{-\frac{1}{2}\ln\left[\tanh\left(\frac{\Omega^2}{\omega_0^2}\right) + \ln\left[\tanh\left(\frac{\beta\Omega}{2}\right)\right]\right]\right\}\right)^2.$$

This section shows that, for mesoscopic RLC circuits with constant parameters, when the Hamiltonian of system is known, the quantum state evolution of the system is determined by the initial state, i.e., the circuit evolves to the generalized coherent state when it is initially in ground state, and to the squeezed state when initially in excited state.

5 Conclusions

In the present work, we analyze the dynamic behavior of a mesoscopic RLC circuit with either constant or time-dependent frequency. In both cases we evaluate the density matrix associated with the dynamics of the mesoscopic circuit. Using the maximum entropy principle approach, we are able to (1) describe coherent and squeezed states as the results of different initial knowledge about the system, which is a direct consequence of including the Hamiltonian as a relevant operator into the density matrix. We also (2) discuss the possibility of describing the dynamical behavior of coherent and squeezed states for both the zero and nonzero-temperature case. Besides, (3) we analyze the dynamical evolution of the quantum states in this paper.

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